- ***Factoring Polynomials***

Computing factors of polynomials requires knowledge of different formulas and some experience to find out which formula to be applied. Below, we give some important formulas:

$x^2 - y^2 = (x + y)(x - y)$

$x^2 + 2xy + y^2 = (x + y)^2$

$x^2 - 2xy + y^2 = (x - y)^2$

$x^3 - y^3 = (x - y)(x^2 + xy + y^2)$

$x^3 + y^3 = (x + y)(x^2 - xy + y^2)$

$a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - ac - bc)$

- ***Completing the Square***

  Derivation

The purpose of "completing the square" is to either factor a prime quadratic equation or to more easily graph a parabola. The procedure to follow is as follows for a quadratic equation $y = ax^2 + bx + c$:

1. Divide everything by a, so that the number in front of x^2 is a perfect square (1):

$$\frac{y}{a} = x^2 + \frac{b}{a}x + \frac{c}{a}$$

2. Now we want to focus on the term in front of the x. Add the quantity $\left(\frac{b}{2a}\right)^2$ to both sides:

$$\frac{y}{a} + \left(\frac{b}{2a}\right)^2 = x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 + \frac{c}{a}$$

3. Now notice that on the right, the first three terms factor into a perfect square:

$$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = \left(x + \frac{b}{2a}\right)^2$$

Multiply this back out to convince yourself that this works.

4. Therefore the *completed square* form of the quadratic is:

$$\frac{y}{a} + \left(\frac{b}{2a}\right)^2 = \left(x + \frac{b}{2a}\right)^2 + \frac{c}{a}$$

or, multiplying through by a,

$$y = a\left(x + \frac{b}{2a}\right)^2 + c - \frac{b^2}{4a}$$

<u>Example</u>

The best way to learn to complete a square is through an example. Suppose you want to solve the following equation for x.

$2x^2 + 24x + 23 = 0$ Does not factor easily, so we complete the square.

$x^2 + 12x + 23/2 = 0$ Make coefficient of x^2 a 1.

$x^2 + 12x = - 23/2$ Add – 23/2 to both sides.

$x^2 + 12x + 36 = - 23/2 + 36$ Take half of 12 (coefficient of x), and square it. Add to both sides.

$(x + 6)^2 = 49/2$ Factor. Now we can take square roots to easily solve this form of the equation.

$\sqrt{(x + 6)^2} = \sqrt{49}/\sqrt{2}$

$x + 6 = 7/\sqrt{2}$

$x = -6 + (7\sqrt{2})/2$ Rationalize the denominator.

- ***<u>Quadratic Equation</u>***

The easiest way to solve a quadratic equation is to factor it. Sometimes these equations do not factor easily, however, so the quadratic formula must be used. A quadratic equation looks like ax^2+bx+c=0, where a, b, and c are just numbers and they are called “numerical coefficients”. Notice that a quadratic equation will always have two values where x=o and therefore that the quadratic equation is plus and minus. You must solve for both in order to get both zeros – where x=o.

<u>Derivation</u>

The solutions to the general-form quadratic function $ax^2 + bx + c = 0$ can be given by a simple equation called the quadratic equation. To solve this equation, recall the *completed square form* of the quadratic equation derived in the previous section:

$$y = a(x + \frac{b}{2a})^2 + c - \frac{b^2}{4a}$$

In this case, $y = 0$ since we're looking for the root of this function. To solve, first subtract c and divide by a:

$$-\frac{c}{a} + \frac{b^2}{4a^2} = (x + \frac{b}{2a})^2$$

Take the (plus and minus) square root of both sides to obtain:

$$\pm\sqrt{-\frac{c}{a} + \frac{b^2}{4a^2}} = x + \frac{b}{2a}$$

Subtracting $\frac{b}{2a}$ from both sides:

$$x = \frac{-b}{2a} \pm \sqrt{-\frac{c}{a} + \frac{b^2}{4a^2}}$$

This is the solution but it's in an inconvenient form. Let's rationalize the denominator of the square root:

$$\sqrt{-\frac{c}{a} + \frac{b^2}{4a^2}} = \sqrt{\frac{-4ac + b^2}{4a^2}} = \frac{\sqrt{b^2 - 4ac}}{2|a|} = \pm\frac{\sqrt{b^2 - 4ac}}{2a}$$

Now, adding the fractions, the final version of the quadratic formula is:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

This formula is very useful, and it is suggested that the students memorize it as soon as they can.

Discriminant

The part under the radical sign, $b^2 - 4ac$, is called the discriminant, Δ. The value of the discriminant tells us some useful information about the roots.

- If $\Delta > 0$, there are two unique real solutions.
- If $\Delta = 0$, there is one unique real solution.
- If $\Delta < 0$, there are two unique, conjugate imaginary solutions.
- If Δ is a perfect square then the two solutions are *rational*, otherwise they are irrational conjugates.